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EVALUATING SHRINKAGE REGRESSION MODELS IN MACROECONOMIC FORCASTING IN THE PRESENCE OF MULTI-COLLINEARITY

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Abstract

The research assesses how four regression models namely Lasso, Ridge, Elastic Net and Best Subset Model perform at economy variable predictions with emphasis on prediction precision and overall stability along with generalization ability. The main evaluation metrics comprise of Mean Squared Error (MSE) and R-Squared together with Adjusted R-Squared and Leave-One-Out Cross-Validation (LOOCV) MSE. Results demonstrate that the Best Subset Model stands out with its exceptional accuracy through MSE measurements of 0.001 and 0.0016 in LOOCV. Alternatively, Ridge Regression shows strength in dealing with multicollinearity problems. Elastic Net maintains a balanced combination of feature selection capabilities with multi-collinearity control while Lasso demonstrates strong feature selection yet it leads to overly-shrunken results. The Best Subset Model stands out as the most dependable predictor for macroeconomic forecasting because it offers the best combination of model complexity and interpretability.

Keywords: Shrinkage regression Models, Macroeconomic Variables, Forecasting, Multicollinearity, Evaluation Metrics.

Introduction

The indicator known as Gross Domestic Product (GDP) represents the principal measurement tool that shows a country's economic state. GDP illustrates the total market worth of all production output from ending products and services during a period that extends past one year and up to quarters (Jones, 2022). Over the years, economists have chosen GDP as their primary tool to monitor national success alongside progress levels and standard of living. The recognized limitations of GDP as a total well-being measurement created new efforts to create better nationwide progress indicators (Provost & Fawcett, 2013). GDP continues to serve as an essential instrument for economic evaluation because it guides the decisions of scholars and investors together with policymakers.

The GDP serves as a primary metric for evaluating economic growth as well as and assessment of living standards. Economic experts inspect GDP measurement precision due to slow productivity growth some years back, which contradicts technical and medical progress (Feldstein, 2017). Many academics have looked into measurement errors that cause these findings while developing new analytical approaches (Moulton, 2018). The formulation of economic policy depends on precise GDP forecasts because Nigeria as a developing economy uses economic growth to fight poverty (Dauda, 2017). The process of GDP projection remains difficult because it is affected by numerous economic variables during its movement. This research employs four different penalized regression techniques for GDP prediction - Lasso, Ridge, Elastic Net and Best Subset regression. The methods differ from each other according to their ability for predictor's selection combined with their model performance optimization and accuracy improvement capability.

However, when multi-collinearity exists among explanatory variables, traditional regression methods like Ordinary Least Squares (OLS) become unstable, leading to inflated standard errors, unreliable coefficient estimates, and poor prediction accuracy (James et al., 2013). In this study, the use of six interrelated macroeconomic predictors; Inflation (LINF), Foreign Exchange (LFE), Foreign Direct Investment (LFDI), Unemployment (LUN), International Trade Statistics (LITS), and Total Expenditure (LTE) raises concerns about multi-collinearity. Shrinkage regression techniques, such as Ridge and Lasso, address this issue by introducing penalty terms that regularize coefficient estimates. Ridge regression penalizes the sum of squared coefficients to reduce variance without eliminating predictors, while Lasso penalizes the absolute values of coefficients, enabling variable selection and improving model interpretability (Tibshirani,1996). These methods help reduce model complexity, limit overfitting, and produce more robust forecasts in high-dimensional, correlated macroeconomic data. According to Kim and Swanson (2018), shrinkage and modern machine learning techniques often outperform traditional models by effectively managing multi-collinearity and capturing complex variable interactions, making them well-suited for reliable macroeconomic forecasting.

Ufoeze et al. (2018) conducted research to determine how exchange rate volatility affects Nigerian economic growth rates. The analysis utilized the dollar-to-naira exchange rate, inflation rate, money supply along with oil revenue as explanatory data to evaluate economic growth through GDP as the response measure. A study using data from CBN Statistical Bulletin confirmed through linear regression that an unregulated exchange rate better evaluated economic growth than an exchange rate controlled with fixed limits.

Alkhateeb and Sultan (2016) conducted a study on export relations with economic expansion in Saudi Arabia. The authors employed ADF, PP unit root tests along with co-integration tests and VECM to establish a mutual connection between exports and economic growth. The research pointed out that Saudi Arabia requires trade policy liberalization to achieve continued economic growth.

Using secondary data from the CBN Statistical Bulletin, Elijah and Musa (2019) investigated the connection between trade openness and Nigeria's economic development between 1980 and 2016. Through unit root, co-integration, and error correction models, they discovered that trade openness impeded economic growth, with imports having a bigger impact than exports. Lipper (2020) examined asset pricing penalization models by contrasting Lasso, Ridge, and Elastic Net regressions. When it came to managing highly correlated variables, Lasso fared better than Ridge, whereas Elastic Net performed worse. The study highlighted Lasso's better prediction accuracy and underlined the significance of model selection.

Li and Chen (2014) conducted a comparison between dynamic factor models and LASSO-based models for the purpose of macroeconomic variable forecasting. Diagnostic factor models provided further enhancement when implemented alongside LASSO because they produced better predictions while creating interpretable predictor groups and achieved superior forecasting accuracy. The research outcomes illustrated that LASSO excels at data-rich environments for both performance improvement and relevant information retrieval.

Tansuchat et al. (2023) studied Thailand's economic statistics spanning from 1970 to 2021 for analyzing the accuracy of GDP predictions through Ridge, Lasso, Elastic Net, as well as SCAD regression models. RPC used eight macroeconomic variables to test different models against each other and determined Elastic Net offered maximum success with an accuracy rate of 99.55%. They used an 80:20 training-testing split. The research outcome helped advance Elastic Net utilization across industries to enhance economic prediction while demonstrating the superiority of complex variable selection above traditional econometric processes. This research evaluates the Lasso and Ridge with Elastic Net along with Best Subset Models to determine their predictive abilities regarding macroeconomic data. Computing macroeconomic prediction models examines multiple metrics to identify the most effective forecasting technique focusing on measurement accuracy together with model stability and data generalization ability.

Materials and Methods

Macroeconomic data from 1995 to 2021 were obtained from the Central Bank of Nigeria Statistical Bulletin and the National Bureau of Statistics. The analysis focuses on LGDP growth as the dependent variable, examined in relation to six predictors: LINF, LFER, LFDI, LUNP, LITS, and LTEX. Shrinkage regression techniques were employed for the analysis using relevant R packages.

2.1 Model Identification and Specification

Regression techniques are widely used across disciplines to examine the relationship between variables and forecast outcomes. In this study, four regression techniques; Lasso, Ridge, Best Subset Selection, and Elastic Net are employed due to their effectiveness in handling multi-collinearity and improving model performance in macroeconomic forecasting.

The dependent variable in the model is LGDP (log of Gross Domestic Product), and the independent variables are: LINF: Log of Inflation, LFER: Log of Foreign Exchange Rate, LFDI: Log of Foreign Direct Investment, LUNP: Log of Unemployment, LITS: Log of International Trade Statistics, LTEX: Log of Total Expenditure. The general form of the specified linear regression model is given as:

$$\mathrm{LGDP}_t = \beta_0 + \beta_1 \mathrm{LINF}_t + \beta_2 \mathrm{LFER}_t + \beta_3 \mathrm{LFDI}_t + \beta_4 \mathrm{LUNP}_t + \beta_5 \mathrm{LITS}_t + \beta_6 \mathrm{LTEX}_t + \varepsilon_t \qquad (1)$$

Where:

β_0 is the intercept, $\beta_1, \ldots, \beta_6$ are the coefficients of the independent variables, ε_t is the error term, t denotes the time period.

2.2 Model Estimation

2.2.1 Ridge Regression Model

Hoerl and Kennard (1970) adopted Ridge Regression to address multi-collinearity, which increases variance in least squares estimates. Ridge Regression minimizes the residual sum of squares (RSS) while adding an L2 penalty ($\lambda \sum_{j=1}^{p} \beta_j^2$) to shrink coefficients and reduce overfitting. It is useful when predictors are highly correlated, preventing large coefficient values but not setting them to zero. The tuning parameter λ controls the trade-off between model complexity and bias-variance tradeoff.

$$\sum_{i=1}^{n} \left(y_i - \beta_0 - \sum_{j=1}^{p} \beta_j x_{ij}\right)^2 + \lambda \sum_{j=1}^{p} \beta_j^2 \qquad (2)$$

$$\min_{\beta} \left\{ \sum_{t=1}^{n} (LGDP_t - L\hat{GDP}_t)^2 + \lambda \sum_{j=1}^{6} \beta_j^2 \right\} \qquad (3)$$

where $\lambda > 0$ is a tuning parameter controlling shrinkage. Setting $\lambda = 0$ yields OLS estimates (Hoerl & Kennard, 1970; Hasan et al., 2016).

2.2.2 Least Absolute Shrinkage and Selection Operator (LASSO Model)

Tibshirani (1996) proposed LASSO, which applies an L_1 norm constraint, shrinking some coefficients to zero:

$$\sum_{i=1}^{n} \left(y_i - \beta_0 - \sum_{j=1}^{p} \beta_j x_{ij}\right)^2 + \lambda \sum_{j=1}^{p} |\beta_j| \qquad (4)$$

$$\min_{\beta} \left\{ \sum_{t=1}^{n} (LGDP_t - L\hat{GDP}_t)^2 + \lambda \sum_{j=1}^{6} |\beta_j| \right\} \qquad (5)$$

LASSO Regression minimizes the residual sum of squares (RSS) while adding an L1 penalty ($\lambda \sum_{j=1}^{p} |\beta_j|$) to encourage sparsity in the model. It can shrink some coefficients to exactly zero, performing both regularization and variable selection. The tuning parameter λ controls the balance between model complexity and sparsity (Tibshirani, 1996; Hamedani & Moosavi, 2017).

2.2.3 Elastic Net Model

Zou and Hastie (2005) employed Elastic Net (ENET) as an improved LASSO. It combines Ridge Regression (L_2) and LASSO (L_1) penalties to improve feature selection and regularization. The mixing parameter α controls the balance between the two, where $\alpha = 0$ reduces to Ridge, and $\alpha = 1$ reduces to LASSO (Hamedani & Moosavi, 2017). It is particularly useful for handling correlated predictors and overcoming LASSO's variable selection limitations.

$$\lambda \left[\frac{1}{2} (1 - \alpha) \sum_{j=1}^{p} \beta_j^2 + \alpha \sum_{j=1}^{p} |\beta_j| \right] \qquad (6)$$

$$\min_{\beta} \left\{ \sum_{t=1}^{n} (LGDP_t - L\hat{GDP}_t)^2 + \lambda \left[\alpha \sum_{j=1}^{6} |\beta_j| + (1 - \alpha) \sum_{j=1}^{6} \beta_j^2 \right] \right\} \qquad (7)$$

2.2.4 Subset Regression Model

Subset Regression fits models with different numbers of regressors and selects the best model using criteria such as R^2_{Adj}, AIC, BIC, C_p and LOOCV. The number of models is:

$$2^{(k-1)} \qquad (4)$$

where k is the number of regressors.

Results and Discussion

Here, the results of the empirical analysis are presented to evaluate the effectiveness of various shrinkage regression techniques; Lasso, Ridge, Elastic

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Net, and Best Subset Selection in forecasting Nigeria's economic growth using some selected macroeconomic indicators.

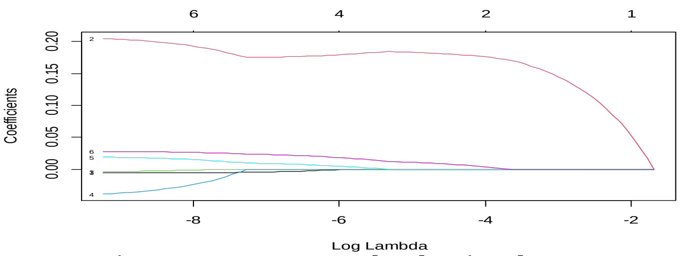

Figure 1: Lasso Trace Plot showing the macroeconomic data against log λ

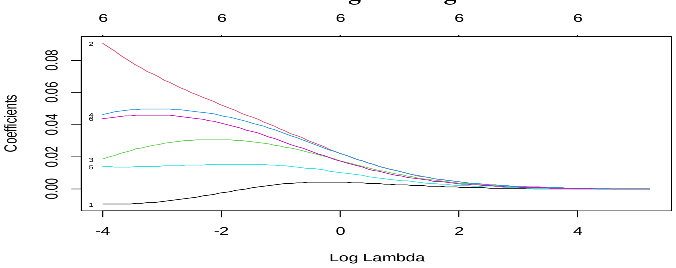

Figure 2: Ridge Trace Plot showing the macroeconomic data against logλ

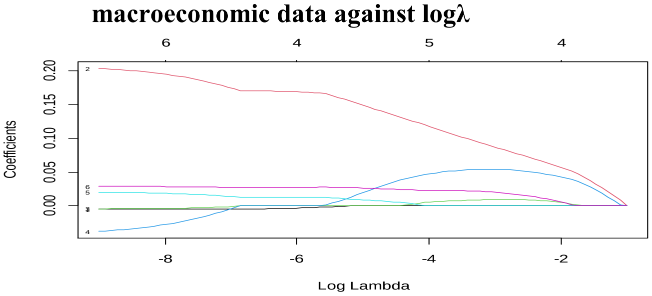

Figure 3: Elastic Net Trace Plot showing the macroeconomic data against log λ

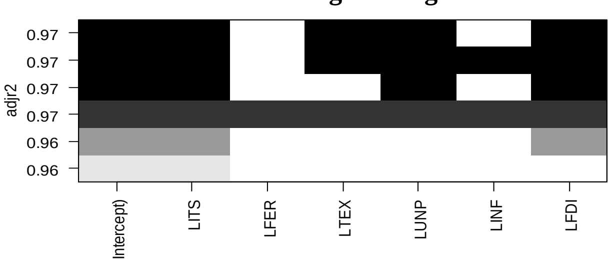

Figure 4: Macroeceomic Feature Selection Plot

Lasso Regression, Ridge Regression, and Elastic Net each handle feature selection and multi-collinearity differently. Lasso Regression (Figure 1) performs feature selection by shrinking some coefficients to zero, highlighting only LITS and LTEX as significant variables influencing the model. Ridge Regression (Figure 2) includes all predictors, shrinking coefficients toward zero without eliminating any, which helps manage multicollinearity and stabilizes the model. Elastic Net (Figure 3) combines the penalties of both Lasso and Ridge, selecting LITS, LFER, LTEX, and LFDI as significant variables with non-zero coefficients, effectively balancing feature selection and multi-collinearity control. Meanwhile, Best Subset (Figure 4) identifies LITS, LTEX, LUNP, and LFDI as the most impactful variables for the model.

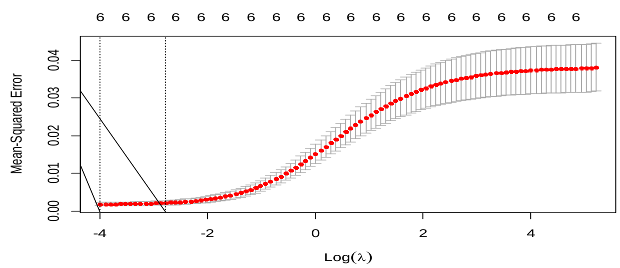

Figure 5: Depicted a LASSO Regression's Cross-validated mean square prediction error against lambda.

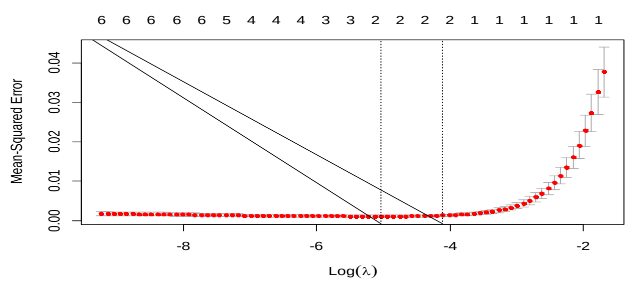

Figure 6: Depicted a RIDGE Regression's Cross-validated mean square prediction error against lambda.

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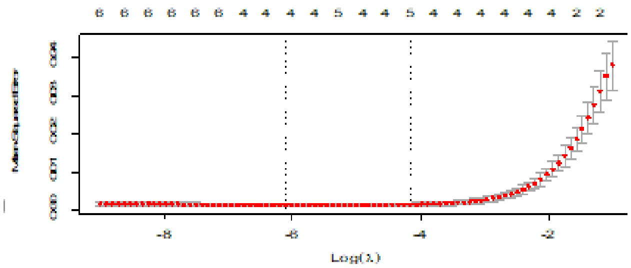

Figure 7: Depicted an ELASTIC NET Regression's Cross-validated mean square prediction error against lambda.

Figures 5, 6, and 7 illustrate the relationship between Mean Squared Error (MSE) and the regularization parameter λ for Lasso, Ridge, and Elastic Net regression models, respectively. In each plot, the MSE initially remains low at small λ values, indicating a good model fit with minimal regularization. However, as λ increases, MSE rises sharply due to over-regularization, leading to underfitting. Vertical dotted lines mark the optimal λ ranges where MSE is minimized, indicating the best trade-off between model complexity and prediction accuracy. The error bars reflect variability in MSE estimates, with greater instability observed at higher λ values. The collection of plots demonstrates how precise λ adjustment enables correct model performance by determining the ideal bias to variance equilibrium.

Table 1: Subset Feature Selection

Subset	LITS	LFER	LTEX	LUNP	LINF	LFDI
1	*					
2	*					*
3	*			*		*
4	*		*	*		*
5	*		*	*	*	*
6	*	*	*	*	*	*

Source: Author's computation using RStudio

The Best Subset selection from the Table under Figure 4 determines LITS, LTEX, LUNP and LFDI as the variables that have the greatest impact on the model. The variables LITS LTEX LUNP and LFDI show up in the different subsets illustrated by table 1 and these key variables drive strong model performance according to Subset 4 data specifically.

Table 2: Model Evaluation Metrics

Metric	Lasso Model	Ridge Model	Elastic Net Model	Best Subset Model
MSE	0.035	0.003	0.015	0.001
R^2	0	0.902	0.559	0.98
Adj. R^2	-1	0.804	0.12	0.97
LOOCV_MSE	0.0039	0.0037	0.0032	0.0016

Source: Author's computation using RStudio

All performance metrics show the best subset model as the dominant choice in table 2. The model produces the most accurate result with a Mean Squared Error (MSE) value of 0.001 which indicates an ideal relationship with the provided data. The regression model delivers powerful explanation because its R-square value reaches 0.980 and its Adjusted R-square reaches 0.970 which shows strong performance despite adjusting for predictor numbers. Its prowess extends to cross-validation, where it maintains the lowest Leave-One-Out Cross-Validation (LOOCV) MSE of 0.0016, indicating outstanding generalizability to new data. In comparison, the Ridge Model also performs well with an MSE of 0.003 and an R-square of 0.902, positioning it as the second-best model overall. Conversely, the Elastic Net Model and Lasso Model show significantly weaker performance, with the Lasso Model exhibiting the highest MSE of 0.035 and an R-square of 0, revealing its poor fit to the macroeconomic data. Moreover, the Lasso Model's negative Adjusted R-square and the highest LOOCV MSE of 0.0039 further highlight its inferior performance relative to the other models.

Table 3: Best Subset Regression Evaluation Results

Variable	Estimate	Std. Error	t value	Pr(>|t|)	VIF
(Intercept)	3.941	0.081	48.744	< 0.0000000000000002 ***	----
LUNP	0.864	0.124	6.961	0.00000033730 ***	3.46
LFDI	0.330	0.035	9.315	0.00000000193 ***	3.82

Source: Author's computation using RStudio

*Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1*

Residual standard error: 0.07788 on 24 degrees of fr eedom

Multiple R-squared: 0.8622, Adjusted R-squared: 0. 8507

F-statistic: 75.07 on 2 and 24 DF, p-value: 0.000000 00004697

In the best subset regression analysis in table 3, the intercept estimate of 3.94107 is highly significant, as evidenced by its substantial t-value of 48.744 and an exceedingly small p-value, indicating it is a crucial part of the model. The predictor LUNP has a coefficient of 0.86434 with a standard error of 0.12416, yielding a t- value of 6.961 and a very small p-value, signifying its strong significance. Similarly, LFDI's coefficient of 0.32979, with a standard error of 0.03540, results in a t-value of 9.315 and an extremely small p-value, showing its significant predictive power. The variables LUNP and LFDI have VIF values of 3.463 and 3.818, respectively. These values fall within the commonly accepted range of 1 to 5, which indicates a moderate level of multi-collinearity. According to Adeboye et al. (2014), multi-collinearity is not considered severe unless VIF values exceed 10, which is the conventional threshold indicating a significant linear dependence among explanatory variables. Therefore, the observed VIFs suggest that while some multi-collinearity exists, it is not at a level that threatens the reliability of the regression estimates. The model's residual standard error of 0.07788 suggests a precise fit, with a Multiple R-squared of 0.8622 indicating that about 86.22% of the variability in the response is explained by the predictors. The Adjusted R-squared of 0.8507 confirms the model's effectiveness while adjusting for the number of predictors, and the high F-statistic of 75.07 with a very small p-value demonstrates that the model overall is highly significant and that the predictors are effectively related to the response variable.

Conclusion

The comparative analysis of Lasso, Ridge, Elastic Net, and Best Subset Regression models for forecasting macroeconomic variables highlights Best Subset Regression as the most effective due to its superior performance metrics, including the lowest Mean Squared Error (MSE) and the highest R-Squared and Adjusted R-Squared values. Its ability to evaluate all subsets of predictor variables ensures optimal selection, reducing the risk of omitting significant predictors and enhancing forecasting accuracy. Ridge Regression also performed well, particularly in handling multi-collinearity by stabilizing the model through a penalty term, making it a robust alternative when predictors are highly interrelated. Elastic Net Regression, while balancing Ridge and Lasso features, did not surpass Ridge or Best Subset Regression in accuracy, suggesting it may require further refinement for macroeconomic forecasting. Lasso Regression, though effective in feature selection, struggled with excessive shrinkage, reducing generalizability and making it the least effective model in this study.

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